Adaptive Control

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Ship-steering, ore-crushing, and telephone switching are examples of operations that require control mechanisms. In these situations the characteristics of the process to be controlled slowly change in time. Adaptive control is the topic in the research area of system and control theory in which control problems for such situations are studied. A synthetic procedure for adaptive control that has been proposed and applied is selftuning control. The purpose of this expository paper is to discuss synthesis and analysis of selftuning controllers.

1. Introduction

Nowadays many production processes and operations in industry require sophisticated control mechanisms. An example is the production of glass tubes in which the wall thickness and the diameter need to be controlled. Similar problems occur in other fields such as the operation of satellites or aircraft. Often it suffices to use a controller, in the form of a computer program, which does not change during its lifetime. Sometimes however one does not know precisely how the process will evolve in the long run, for example when steering a ship under varying conditions such as the weather. In such a case it is necessary to let the controller adapt continuously to the new situation. In a 4-year project, concluded in 1987, several aspects of adaptive control theory were studied at the CWI.

2. Motivation of adaptive control

Control problems arise in, for example, electrical or mechanical engineering. A specific example is the motion of a robot arm. An engineer faced with the problem of designing a controller first develops an engineering model, the *plant.* From this he extracts a mathematical model, called a *dynamical system,* or just a *system.* A control signal, called the *input,* may influence the plant, for example the signal for a motor driving the robot arm. Measurements performed on the robot arm are called *outputs.*

The *control problem* may then be formulated as follows: construct an input such that if this input is applied to the dynamical system, the behaviour of the system satisfies certain control objectives. In the example of the robot arm a function is to grasp and move an object. In this task the control objective could be that it is done with minimal energy and in a smooth way. Usually the input is based on the output of the dynamical system, and the function that maps outputs to inputs is called a *feedback law* or, if it is a dynamical system, a *compensator.* In either case one calls it a *controller.* Several procedures have been developed to synthesize controllers. One approach is to formulate an optimization criterion, like the energy used to move the robot arm that is to be minimized, and then determine a controller that will achieve it. The controller may be programmed on a computer and is connected to the object to be controlled, say the robot arm. The arm is now continuously controlled and is then said to operate in *closed-loop.* Controllers designed in this way work satisfactorily in widely different settings, such as windmills, steam boilers and electricity networks.

However, a limitation of this approach shows up if there is uncertainty about the process to be controlled, or if the characteristics of the process slowly change in the course of time. For example, the steering of a ship in quiet sea is rather different from that in rough sea. Similarly, control of the flows of material in an ore crusher should be adjusted on the basis of ore size. Another example is a robot arm that has to lift objects of varying weights. Here an approach such as the one mentioned above does not in general yield a practical solution. The controller should be able to adapt itself continuously to the characteristics of the process. An ideal adaptive controller would be able to control a dynamical system on the basis of observations only and hence could be connected to every dynamical system. However, it is very doubtful whether this ideal will ever be reached. Anyway, research activities in adaptive control have increased tremendously during the last fifteen years, and successful applications have been reported in connection with ship-steering and ore-crushing. The implementation of adaptive controllers has become feasible due to the availability of relatively cheap, fast and reliable digital computers.

3. Selftuning control

An important and well-known synthesis procedure of adaptive control is *selftuning control.* It was introduced by K.J. Aström and B. Wittenmark in [2]. A selftuning controller is a combination of a recursive parameter estimator and a controller. These concepts are described below.

A dynamical system is a mathematical model for a phenomenon. Consider the behaviour of a ship at sea. An engineering model of such a ship, the plant, consists of a mass that is affected by the waves. The dynamical system that models the ship's movements may then be derived by use of the laws of mechanics, and by use of a stochastic model for the wave spectrum. In general, for a phenomenon to be modelled one proposes a class of systems that is called the *model class.* The elements in this class are described by a *parametrization.* The variables in this parametrization are called *parameters.* In the example of a ship, parameters are the mass of the ship and the dominating frequencies in the wave spectrum. For later use an elementary example of a dynamical system is introduced by the representation

$$x_{t+1} = ax_t + bu_t,\ x_0, \tag{1}$$

$$y_t = x_t, \tag{2}$$

in which x_t is the state at time $t \in T = \{0,1,...\}$, $x: T \to \mathbb{R}$, u_t the input at time t, y_t the output at time t, and $a,b \in \mathbb{R}$ are the parameters. This is called a *discrete-time, first order, finite-dimensional, linear system.*

Modelling the behaviour of a ship at sea can now be phrased as: how can one fit a dynamical system to observed data? Suppose that the engineer has selected a model class and a parametrization of this model class. The problem of fit can then be rephrased as: determine a system in the model class that best fits the data. Solution of this question requires a measure for misfit and a measure for complexity of systems in the model class. The *misfit* is the difference between the data and what the model specifies about the observations. A model that always models the data exactly is all of the data. But this model is rather complex. Hence a *measure of complexity,* say the order of a system, is necessary. Once these measures are defined, one can determine a system in the model class that minimizes a combination of the misfit and the complexity. One way to perform this minimization is to determine those parameter values that minimize a likelihood criterion. Such a procedure is called *parameter estimation.* This topic, that forms part of the research area of *system identification,* will not be explored here any further; for a reference see [13].

Adaptive control is intended for control problems in which the object to be modelled is constantly changing. Therefore there is a need for a model that is appropriate for all circumstances. By continuously adapting the parameter values of a dynamical system, one may achieve this objective. An algorithm that continuously estimates the parameters in a recursive way is called a *recursive parameter estimator.* It is indicated in section 5 how one can construct a parameter estimator. For the mathematical example introduced above and the parameters $a,b \in \mathbb{R}$ a recursive parameter estimator is given by

$$\hat{a}(t+1) = \hat{a}(t) + \frac{y(t)}{y^2(t)+u^2(t)}[y(t+1)-\hat{a}(t)y(t)-\hat{b}(t)u(t)], \tag{3}$$

$$\hat{b}(t+1) = \hat{b}(t) + \frac{u(t)}{y^2(t)+u^2(t)}[y(t+1)-\hat{a}(t)y(t)-\hat{b}(t)u(t)]. \tag{4}$$

The control part of a selftuning controller is derived as in regular control synthesis. Given a dynamical system and a control objective, a control law can be derived. Consider again the above introduced example and the control objective of *pole placement.* (The notion of a *pole* will not be formally defined here. It is illustrated in the example given below.) Suppose that the pole must be placed at the value $c \in \mathbb{R}$. The input should then be taken as

$$u_t = \frac{c-a}{b} y_t \tag{5}$$

and the feedback law as

$$f(a,b) = \frac{c-a}{b}. \tag{6}$$

This feedback law applied to the system (1,2) yields the closed-loop system

$$x_{t+1} = cx_t,\ x_0, \tag{7}$$

$$y_t = x_t. \tag{8}$$

Thus the pole of the system is placed at the value c. Note that the feedback law is dependent on the parameters a,b of the system.

The *synthesis procedure of selftuning control* now prescribes that at any time:

- one estimates the values of the parameters given available observations of inputs and output;
- one applies a feedback law assuming that the current parameter estimates are the true ones.

At any time instant this cycle is repeated. For the example used above, a selftuning controller is then given by the recursive parameter estimator (3,4) and the input

$$u_{t+1} = f(\hat{a}_{t+1},\hat{b}_{t+1})\, y_{t+1}. \tag{9}$$

A selftuning controller is thus based on a simple cybernetic principle. It can be derived by combining well-known results of control theory and system identification. Applications of selftuning controllers showed that they work surprisingly well in widely varying control situations.

Once a selftuning controller had been proposed and applications had established its usefulness, theoretical questions were formulated. Selftuning control leads to the following questions: Does a selftuning controller satisfy the original control objective? What is the asymptotic behaviour of a system controlled by a selftuning controller? A selftuning controller consists of a combination of a particular recursive parameter estimator and a particular feedback law. Hence the question, which recursive parameter estimator should be combined with which feedback law?

An analysis of selftuning control algorithms turned out to be quite difficult. The first result, proven in [2], applies to stochastic systems with a particular selftuning controller. It says that if the system generating the observations is in the model class, and if the parameter estimates converge to the true values, then the resulting controller is asymptotically the same as that associated with the true parameter values. Further progress was made by P. Varaiya and V. Borkar [6]. They considered selftuning control for a finite-state Markov chain, since reduced complexity facilitates the analysis. They showed that even if the parameter estimates converge, they may not converge to the true values. In a later paper P.R. Kumar and co-workers [4] showed for a specific selftuning controller that even if the parameter estimates do not converge to the true values, the controller may be asymptotically identical to the one associated with the true parameter values.

4. Closed-loop identification in selftuning control

From the above quoted investigations it was clear that the issue of closed-loop identification in selftuning control needed proper attention. J.W. Polderman, in a 4-year project on adaptive control at CWI, has explored this question. Below are summarized some of his findings, see [14].

The main problem is this: does a selftuning controller lead asymptotically to the controller that would have been used if the parameter values of the system generating the data had been known? If the answer to this question is positive then the selftuning controller is said to be *selftuning*. Note that the usage of this term is such that a selftuning controller may or may not be selftuning. The above mentioned problem can be investigated by way of the following two questions.

The first question is: can all the parameters be determined if parameter estimation takes place in closed-loop? To discuss this question some terminology and notation is introduced. Consider again the example of Section 3. Replace x by y. Assume that the system that generates the data is in the model class, and that it is represented by the parameter values a_0, b_0, called the *true parameter values.* The system is then represented by

$$y_{t+1} = a_0 y_t + b_0 u_t. \tag{10}$$

The input for the pole placement objective and associated with the true parameter values is

$$u_t = \frac{c - a_0}{b_0} y_t. \tag{11}$$

Consider a selftuning controller for the system (10). Assume that the controller has been operating for a while and that the parameter estimates have converged to the values a, b. The input associated with these values is

$$u_t = \frac{c - a}{b} y_t. \tag{12}$$

The resulting closed-loop system is

$$y_{t+1} = [a_0 + b_0 \frac{c - a}{b}] y_t. \tag{13}$$

However, the engineer designing this system is unaware of the values of a_0, b_0. The engineer therefore supposes that the dynamical system is

$$y_{t+1} = a y_t + b u_t, \tag{14}$$

because the values a, b are known to him from the recursive parameter estimator. This system, combined with the feedback law (12), yields the closed-loop system

$$y_{t+1} = c y_t. \tag{15}$$

Because the output y is observed, the equations (13) and (15) lead to the conclusion that

$$a_0 + b_0 \frac{c-a}{b} = c,$$

or

$$\frac{c-a_0}{b_0} = \frac{c-a}{b}. \tag{17}$$

Let

$$\underline{G} = \{(a,b) \in \mathbb{R}^2 \mid \frac{c-a_0}{b_0} = \frac{c-a}{b}\}.$$

Then $\underline{G}$ contains all parameter values that cannot be distinguished from the pair (a_0,b_0) when selftuning control is used. Note that in the above example, in which a_0, b_0 and c are fixed, $\underline{G}$ is a line in the plane. The conclusion is thus that recursive parameter estimation in the presence of feedback may lead to parameter values that differ from the true parameter values. The implications of this conclusion for selftuning control are explored below. For other classes of systems the set $\underline{G}$ may be characterized by, for example, rational functions. Its analytic form may be hard to describe explicitly.

The second question is: which parameter values yield the same control law as the parameter values associated with the system generating the data? Consider again the above example. The control law associated with the system generating the data is

$$f(a_0,b_0) = \frac{c-a_0}{b_0}, \tag{18}$$

while that associated with the parameter values (a,b) is

$$f(a,b) = \frac{c-a}{b}. \tag{19}$$

Define the set

$$\underline{H} = \{(a,b) \in \mathbb{R}^2 \mid f(a,b) = f(a_0,b_0)\}.$$

Thus $\underline{H}$ is the set of all parameter values that lead to the same feedback law as the true parameter values. In the above example $\underline{H}$ is a line in the plane. With this notation the closed-loop identification issue in adaptive control can be explored.

The main problem of this section, about the asymptotic behaviour of selftuning controllers, can now be formulated mathematically as whether the relation

$$\underline{G} \subset \underline{H} \tag{20}$$

holds. Suppose that in selftuning control the parameter estimates converge to the values (a,b). As argued before $(a,b) \in \underline{G}$, hence these values cannot be distinguished from (a_0,b_0). However, if (20) holds then $(a,b) \in \underline{H}$ and the selftuning control law associated with (a,b) is identical to the control law associated with the true parameter values (a_0, b_0). Hence relation (20) implies that selftuning holds.

The problem now is whether inclusion (20) holds. A modification of (20) can be proved for the *pole placement* control objective applied to discrete-time, single-input-single-output, finite-dimensional, linear systems. The modification says that if the outputs of the systems (10) and (14) are identical, then the inputs (11) and (12) are identical. In the example given above, it follows from (17,18,19) that $G=H$. Therefore one can expect that selftuning control based on this control objective works quite well. Another control objective is *linear-quadratic control* in which for a linear system the quadratic cost function

$$\sum_{s=0}^{t_1} [qy_s^2 + ru_s^2]$$

with q, $r \in (0, \infty)$, is to be minimized. For this control objective one can show that the intersection $G \cap H$ is a negligible set of G. *Negligible* here means that the set $G \cap H$ is an embedded manifold of a strictly smaller dimension than that of G. Therefore selftuning control based on this objective will almost always not yield selftuning.

A related problem is: for which control objectives, or feedback laws, does one have that

$$\underline{G} \subset \underline{H}?$$

For first order systems and under certain smoothness conditions, the answer to this question is that only the pole placement control objective satisfies this condition. For systems of order higher than one this question is not yet settled completely. On the basis of partial results, the conjecture is that the same conclusion holds. With hindsight this conclusion is not that surprising to experts in the field. Pole placement is a control objective that can be verified from the inputs and outputs of a controlled system. This is not true for linear-quadratic control.

The conclusion of this section is that for the selftuning control synthesis procedure defined in Section 3, selftuning control by pole placement achieves selftuning. At least for first order systems, pole placement is the only control objective that leads to selftuning.

5. Adaptive control algorithms

Selftuning control consists of a recursive parameter estimator and a controller. It has been shown in the previous section that if the pole placement control objective is used and if the parameter estimates converge, then the limiting feedback law equals the one associated with the true parameter values. The question is now: how to synthesize a recursive parameter estimator that will produce converging parameter estimates?

A recursive parameter estimator may be designed by using orthogonal projection. Given a parameter estimate, the next parameter estimate may be determined as the projection of the current parameter estimate on a plane in the parameter space determined by the observations. This plane passes a fortiori through the true parameters. By this orthogonal projection construction, the parameter estimates in every step of the recursion get closer to the true parameter values. It can be proved that they will eventually converge to the set G.

Special care should be taken so that the parameter estimates lie inside the subset of the parameter space on which the function, that assigns to every parameter value a controller, is defined. In the example, the feedback law

$$f(a,b) = \frac{c-a}{b} \tag{21}$$

is defined only if $b \neq 0$. To ensure convergence, most known adaptive control algorithms require a condition of the form $|b| > \epsilon$ for some $\epsilon > 0$. For the class of systems considered, a fairly general method was developed in order to modify an algorithm in such a way that the estimates always belong to the required subset of the parameter space. For the example only the condition $b \neq 0$ is needed for convergence. The method is too involved to be reproduced here.

6. Conclusion and open problems

The main contribution of [14] is an analysis of the limitations of selftuning control due to the fact that parameter estimation takes place in closed-loop. A selftuning controller for adaptive pole placement and an adaptive controller for linear quadratic control have been proposed and analyzed.

Several open problems in adaptive control require further attention. The selftuning synthesis procedure must be investigated also for stochastic systems, and for other classes of dynamical systems. If the property of selftuning does not hold, then modifications of the selftuning synthesis procedure must be considered. Adding excitation to the input process is such a modification. Not only asymptotic properties, but also transient properties of adaptive controllers are of interest. Questions are: what is the convergence speed of adaptive control algorithms? How can one improve the tracking ability of algorithms for slowly changing parameters? *Active learning* of the parameters should be explored. In this approach the input process is used to improve the quality of future parameter estimates. A totally different question is: what happens in adaptive control if the system generating the data is not in the model class?

Another open problem is to propose a criterion that can be applied in deciding when to apply adaptive control and when to apply robust control. In robust control, a fixed or non-adaptive controller is to be determined that will achieve a control objective for a specified model class with a set of parameter values.

7. Further reading

Material on system theory may be found in the books [7, 9] and in the papers [16-18]. Books on control theory are [3, 12].

Adaptive control at an introductory level is presented in the books [3, 8, 11, 13]. Survey papers are [1, 10]. Applications are reported in [1, 5, 15].

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